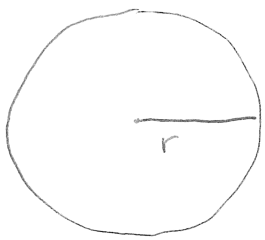
AP Calculus AB
Related Rates Notes

Name: Key

1) An oil tanker strikes an iceberg and has a hole ripped open on its side. Oil is leaking out in a near circular shape. The radius of the oil spill is changing at a rate of 1.5 miles per hour. How fast is the area of the oil spill changing when the radius is 0.6 mile?



$\frac{dr}{dt} = 1.5 \text{ mi/hr}$

$\frac{dA}{dt} = ?$

$r = .6 \text{ mi}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi (.6)(1.5)$

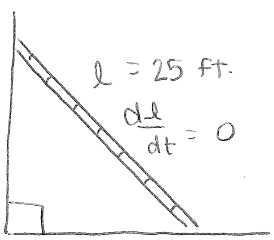
$\frac{dA}{dt} = 1.8\pi \text{ mi}^2/\text{hr}$

2) A 25-foot ladder is leaning against a wall. The bottom of the ladder is being pulled away from the wall at a rate of 3 feet per second.

a) How fast is the top of the ladder moving at the instant the bottom of the ladder is 15 feet away from the wall?

$y =$

$\frac{dy}{dt} = ?$



$\ell = 25$ ft.

$\frac{d\ell}{dt} = 0$

$x = 15$ ft.

$\frac{dx}{dt} = 3$ ft/sec

$15^2 + y^2 = 25^2$

$y = 20$ ft.

$x^2 + y^2 = \ell^2$

$\cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = \cancel{2}\ell \frac{d\ell}{dt}$

$(15)(3) + (20)\frac{dy}{dt} = (25)(0)$

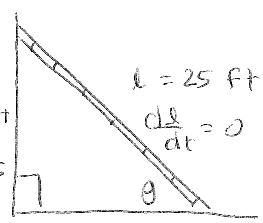
$45 = -20 \frac{dy}{dt}$

$\frac{dy}{dt} = -2.25$ ft/sec

b) How fast is the angle between the ladder and the ground changing at the instant the base of the ladder is 15 feet from the wall?

$y = 20$ ft

$\frac{dy}{dt} = -2.25$



$\ell = 25$ ft

$\frac{d\ell}{dt} = 0$

$x = 15$ ft

$\frac{dx}{dt} = 3$ ft/sec

$\frac{d\theta}{dt} = ?$

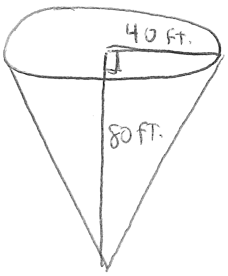
$\sin\theta = \frac{y}{\ell}$

$\cos\theta \frac{d\theta}{dt} = \frac{\left(\frac{dy}{dt}\right)(\ell) - (y)\left(\frac{d\ell}{dt}\right)}{\ell^2}$

$\left(\frac{15}{25}\right)\frac{d\theta}{dt} = \frac{(-2.25)(25) - (20)(0)}{(25)^2}$

$\frac{d\theta}{dt} = -.15$ rad/sec

3) A water authority is filling an inverted conical water storage tower at a rate of 9 cubic feet per minute. The height of the tank is 80 feet and the radius at the top is 40 feet. How fast is the water level inside the tank changing when the water level is 60 feet deep?



$\frac{dV}{dt} = 9 \ ft^3/min$

$\frac{dh}{dt} = ?$

$h = 60$ ft.

$\frac{r}{h} = \frac{40}{80}$

$80r = 40h$

$r = \frac{h}{2}$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{h^2}{4} \cdot h$

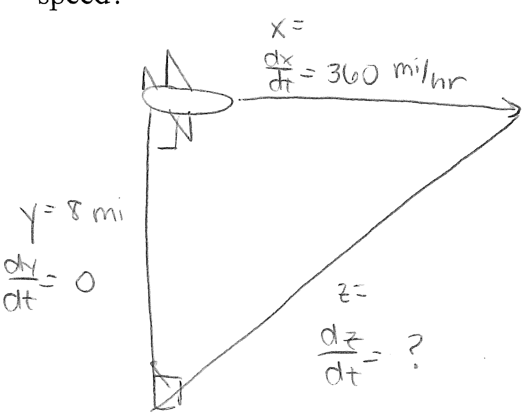
$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$(9) = \frac{\pi}{4}(60)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = .003$ ft/min

4) An airplane is flying at an altitude of 8 miles and passes directly over a radar antenna at a speed of 360 mph. 1 minute later, the radar detects the speed at which the plane is moving away from the radar. What was that speed?



$d = rt$

$d = (360)\left(\frac{1}{60}\right)$

$d = 6$ mi

$6^2 + 8^2 = z^2$

$z = 10$ mi

$x^2 + y^2 = z^2$

$\cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = \cancel{2}z \frac{dz}{dt}$

$(6)(360) + (8)(0) = (10)\frac{dz}{dt}$

$\frac{dz}{dt} = 216$ mi/hr